1) Find a formula for the following sequences

a) $3, -1, -5, -9, -14, \ldots$

$a_n = 3 - 4n$ for $n \geq 0$

OR

$a_n = 7 - 4n$ for $n \geq 1$

OR

$a_0 = 3, \quad a_{n+1} = a_n - 4$

OR

any other correct answer

b) $1, -1, \frac{1}{2}, \frac{-1}{6}, \frac{1}{24}, \frac{-1}{108}$

$a_n = \frac{(-1)^n}{n!}$ for $n \geq 0$

OR

$a_0 = 1, \quad a_{n+1} = a_n \cdot \left(\frac{-1}{n+1}\right)$

OR

any other correct answer

c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$a_n = \frac{n}{n+1}$ for $n \geq 1$

OR

$a_1 = \frac{1}{2}$, $a_{n+1} = \frac{n}{(a_n)(n+2)}$

OR

any other correct answer

d) $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$

$a_n = 3 \cdot \left(\frac{1}{2}\right)^n$ for $n \geq 0$

OR

$a_0 = 3$, $a_{n+1} = \frac{1}{2} \cdot a_n$

OR

any other correct answer

(2) Can you give an example of a sequence which...

a) ... is increasing after 0 and converges to a negative limit?

$a_n = -1 - \frac{1}{n}$ for $n \geq 1$

b) ... is increasing after 0 and diverges to $-\infty$?

No - any increasing sequence is bounded from below and cannot diverge to $-\infty$.

c) is monotone after 0 and diverges to $-\infty$?

$a_n = -n$ for $n \geq 0$

d) ... is monotone after 0, does not converge, does not diverge to ∞, and does not diverge to $-\infty$?

No - if a sequence is monotone increasing, but doesn't diverge to ∞, it must be bounded above. It also must be bounded below because it is increasing. By the Monotone Convergence Theorem, the sequence must converge.

You can make a similar argument in the case that a sequence is monotone decreasing.

e) ... is bounded, but does not converge?

$$a_n = (-1)^n \text{ for } n \geq 0$$

f) ... is decreasing and bounded from below?

$$a_n = \left(\frac{1}{2}\right)^n \text{ for } n \geq 0$$